Midterm Cheatsheet

Bayes' Rule for Events

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A|B)P(B)}{P(A)}$

where, by the Law of Total Probability,

$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$

Posterior Model

$f(\pi|x) = \frac{f(\pi)L(\pi|x)}{f(x)} \propto f(\pi)L(\pi|x)$

Beta Model

$\pi \sim \text{Beta}(\alpha, \beta)$

The Beta model is specified by continuous pdf

$f(\pi) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\pi^{\alpha-1}(1-\pi)^{\beta-1} \quad \text{for } \pi \in [0,1], \alpha > 0, \text{and } \beta > 0$

where $\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx$ and $\Gamma(z+1) = z\Gamma(z)$. Fun fact: when z is a positive integer, then $\Gamma(z)$ simplifies to $\Gamma(z) = (z-1)!$.

Beta Descriptives

$E(\pi) = \frac{\alpha}{\alpha+\beta}$

$\text{Mode}(\pi) = \frac{\alpha-1}{\alpha+\beta-2}$

$\text{Var}(\pi) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

The Beta-Binomial Model

Let $\pi \sim \text{Beta}(\alpha, \beta)$ and $X|n \sim \text{Bin}(n, \pi)$ then

$\pi|(X = x) \sim \text{Beta}(\alpha + x, \beta + n - x)$

Gamma Prior

$\lambda \sim \text{Gamma}(\alpha, \beta)$ where $\alpha > 0$ and $\beta > 0$:

The Gamma distribution is specified by continuous pdf $f(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda} \quad \text{for } \lambda \in [0, \infty)$

Gamma Descriptives

$E(\lambda) = \frac{\alpha}{\beta}$

$\text{Mode}(\lambda) = \frac{\alpha-1}{\beta}$ where $\alpha \geq 1$

$\text{Var}(\lambda) = \frac{\alpha}{\beta^2}$

Poisson Likelihood

$f(x|\lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \quad \text{for } x \in \{0, 1, 2, \ldots, n\}$

The Gamma-Poisson Model

If $f(\lambda) \sim \text{Gamma}(\alpha, \beta)$

and if $x_i \sim iid$ Poissson(λ)for $i \in 1, \ldots, n$

then $f(\lambda|\vec{x}) \sim \text{Gamma}(\alpha + \sum x_i, \beta + n)$.

Normal Prior

If $\theta \sim \text{Normal}(\mu, \tau^2)$ then

$f(\theta) = \frac{1}{\sqrt{2\pi\tau^2}} \exp\{-\frac{1}{2}(\frac{\theta-\mu}{\tau})^2\}$

Normal Likelihood

If $X \sim \text{Normal}(\theta, \sigma^2)$

$L(\theta|\vec{x}) \propto \exp\{-\frac{1}{2}(\frac{\bar{x}-\theta}{\sigma/\sqrt{n}})^2\}$

The Normal Posterior

$\theta|\vec{x} \sim \text{Normal}(\frac{\sigma^2\mu+\tau^2n\bar{x}}{n\tau^2+\sigma^2}, \frac{\sigma^2\tau^2}{n\tau^2+\sigma^2})$

