

1 Review

```
model_mpg <- lm(mpg ~ hp + am + wt, data = mtcars)
```

```
summary(model_mpg)
```

```
##
## Call:
## lm(formula = mpg ~ hp + am + wt, data = mtcars)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.4221 -1.7924 -0.3788  1.2249  5.5317
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 34.002875   2.642659   12.867 2.82e-13 ***
## hp          -0.037479   0.009605   -3.902 0.000546 ***
## am           2.083710   1.376420    1.514 0.141268
## wt          -2.878575   0.904971   -3.181 0.003574 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.538 on 28 degrees of freedom
## Multiple R-squared:  0.8399, Adjusted R-squared:  0.8227
## F-statistic: 48.96 on 3 and 28 DF,  p-value: 2.908e-11
```

```
anova(model_mpg) #anova stands for analysis of variance
```

```
## Analysis of Variance Table
```

```
##
```

```
## Response: mpg
```

```
##           Df Sum Sq Mean Sq F value    Pr(>F)
## hp           1  678.37   678.37  105.354 5.395e-11 ***
## am           1  202.24   202.24   31.408 5.335e-06 ***
## wt           1   65.15    65.15   10.118 0.003574 **
## Residuals  28  180.29     6.44
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
678.37 + 202.24 + 65.15 + 180.29
```

```
## [1] 1126.05
```

```
mtcars %>%  
  mutate(error = mpg - mean(mpg)) %>%  
  mutate(error_sq = error^2) %>%  
  glimpse()
```

```
## Observations: 32
```

```
## Variables: 13
```

```
## $ mpg      <dbl> 21.0, 21.0, 22.8, 21.4, 18.7, 18.1, 14.3, 24.4, 22.8,...  
## $ cyl      <dbl> 6, 6, 4, 6, 8, 6, 8, 4, 4, 6, 6, 8, 8, 8, 8, 8, 8, 4,...  
## $ disp     <dbl> 160.0, 160.0, 108.0, 258.0, 360.0, 225.0, 360.0, 146....  
## $ hp       <dbl> 110, 110, 93, 110, 175, 105, 245, 62, 95, 123, 123, 1...  
## $ drat     <dbl> 3.90, 3.90, 3.85, 3.08, 3.15, 2.76, 3.21, 3.69, 3.92,...  
## $ wt       <dbl> 2.620, 2.875, 2.320, 3.215, 3.440, 3.460, 3.570, 3.19...  
## $ qsec     <dbl> 16.46, 17.02, 18.61, 19.44, 17.02, 20.22, 15.84, 20.0...  
## $ vs       <dbl> 0, 0, 1, 1, 0, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1,...  
## $ am       <dbl> 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,...  
## $ gear     <dbl> 4, 4, 4, 3, 3, 3, 3, 4, 4, 4, 4, 3, 3, 3, 3, 3, 3, 4,...  
## $ carb     <dbl> 4, 4, 1, 1, 2, 1, 4, 2, 2, 4, 4, 3, 3, 3, 4, 4, 4, 1,...  
## $ error    <dbl> 0.909375, 0.909375, 2.709375, 1.309375, -1.390625, -1...  
## $ error_sq <dbl> 0.8269629, 0.8269629, 7.3407129, 1.7144629, 1.9338379...
```

```
mtcars %>%  
  mutate(error = mpg - mean(mpg)) %>%  
  mutate(error_sq = error^2) %>%  
  summarize(sum(error_sq))
```

```
## sum(error_sq)  
## 1 1126.047
```

```
summary(model_mpg)$r.squared
```

```
## [1] 0.8398903
```

```
(678.37 + 202.24 + 65.15)/(678.37 + 202.24 + 65.15 + 180.29)
```

```
## [1] 0.8398917
```

2 Binomial Likelihood Example

You have a friend who claims that they can identify caffeinated and non-caffeinated coffee. Let their success rate be an unknown parameter p . You do a double blind test with 10 cups of coffee. Your friend identifies 8 cups of coffee correctly. Which of the following is the most likely estimation for p ?

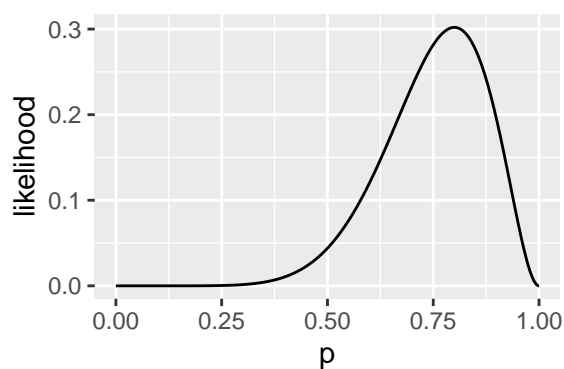
- a) 0.3
- b) 0.5
- c) 0.8

The answer may seem intuitive but let's see the reasoning behind it

```
p <- seq(from = 0, to = 1, by = 0.001)
likelihood <- dbinom(x = 8, size = 10, prob = p)

data <- as.data.frame(cbind(p, likelihood))

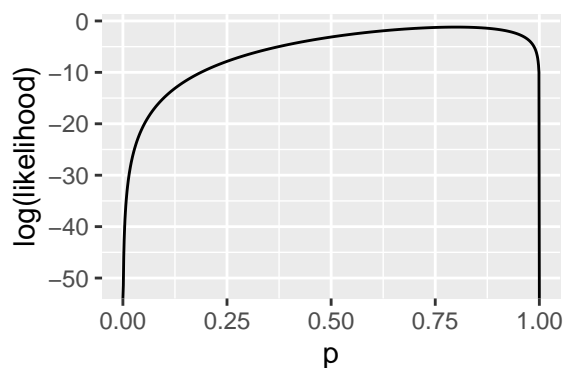
data %>%
  ggplot(aes(x = p, y = likelihood)) +
  geom_line()
```



```
data %>%
  filter(likelihood == max(likelihood))
```

```
##      p likelihood
## 1 0.8 0.3019899
```

```
data %>%
  ggplot(aes(x = p, y = log(likelihood))) +
  geom_line()
```



```
data %>%
  filter(log(likelihood) == max(log(likelihood)))
```

```
##      p likelihood
## 1 0.8 0.3019899
```

Estimating p is quite easy when we are only testing one friend. We are now going to consider two scenarios.

The experiment is repeated with a friend who tests 50 cups of coffee and correctly identifies 40 cups. What is the estimate for p ?

In second scenario:

You repeat the experiment with five friends where each friend tests 10 cups of coffee. Two friends are correct about 7 cups, 1 friend is correct about 8 cups, 2 friends are correct about 9 cups. What is the estimate for p ?

3 Binomial Likelihood Using Calculus