

# 1 Foundations of Probability

## 1.1 Random Process

Definition:

Example:

Note:

## 1.2 Sample Space

Definition:

Example:

### **1.3 Event**

Definition:

Example:

### **1.4 Mutually Exclusive or Disjoint Events**

Definition:

Example:

## 1.5 Defining Probability

## 1.6 Axioms of Probability

1. The probability of any event is between zero and one:

$$0 \leq P(A) \leq 1$$

2. The probabilities must add up to 1.

$$P(\Omega) = 1$$

3. The probability of mutually exclusive events is additive.

$$\bigcup_{i=1}^{\infty} A_i = \sum_{i=1}^{\infty} P(A_i)$$

### Addition Rule

### Law of Complementary Events

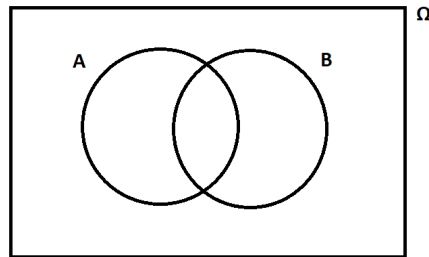
## 2 Representing Events Using Set Notation

### 2.1 Notation

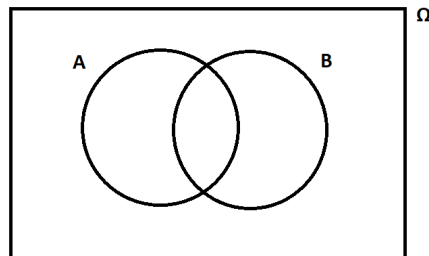
Description	How it is read	How it is written
Union of Sets	in A or in B	$A \cup B$
Intersection of Sets	in A and in B	$A \cap B$

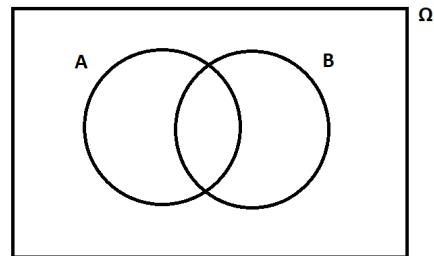
### 2.2 Venn Diagrams

#### Union



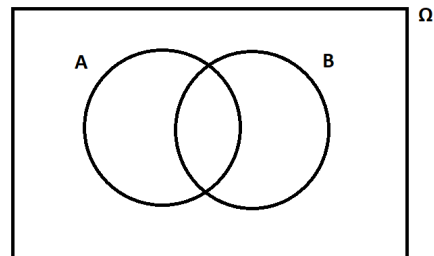
#### Intersection



**Complement**

**Exercise** Please shade the following:

$$(A \cup B^c)^c$$

**Exercise**

Draw a Venn diagram of two disjoint events

### 3 Conditional Probability

The General Social Survey (GSS) is sociological survey that has been regularly conducted since 1972. It is a comprehensive survey that provides information on experiences of residents of the United States. The contingency table below is built off two questions on the survey. The first question asks the respondent's race and the second question asks "Are there any situations you can imagine in which you would approve of a policeman striking an adult male citizen?". The answers to this question is displayed as the `polhitok` variable.

polhitok	race			<b>Total</b>
	White	Black	Other	
Yes	23,260	2,359	1,010	26,629
No	6,969	2,405	1,133	10,507
<b>Total</b>	30,229	4,764	2,143	37,137

1. What is the probability that a randomly selected person in this sample is White?
2. What is the probability that a randomly selected person in this sample is white and approves police strike?
3. What is the probability that a randomly selected person in this sample approves of police strike given that they are White?

### **3.1 Definitions**

**Marginal Probability**

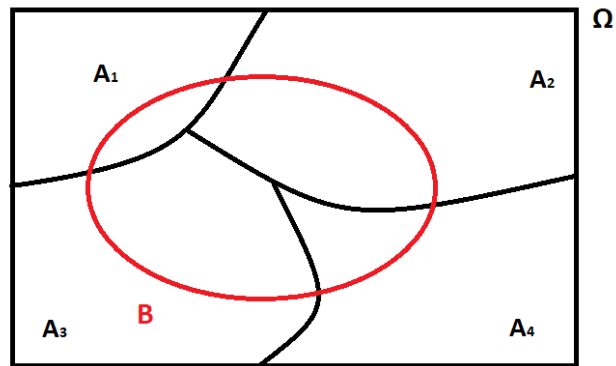
**Joint Probability**

**Conditional Probability**

**Independence**

**Multiplication Rule for Independent Processes**

## 4 Law of Total Probability



## 5 Bayes' Rule for Events



**Exercise**

An algorithm is written to detect cat images. When given a cat image the algorithm identifies the image as a cat image 80% of the time. However, when given an image without a cat the algorithm falsely identifies it as a cat image 50% of the time. The algorithm is to be tested with a set of images of which are 7% cat images. What is the probability that an image is a cat image if the algorithm identifies it as a cat image?