

1 Inference for a Single Proportion

1.1 Conditions

The sampling distribution of a proportion \hat{p} , from a population with a proportion parameter p , based on sample size n is normally distributed if the following conditions are met.

1. The sample data are independent.
2. There needs to be at least 10 successes and 10 failures in the sample.
3. The sample size is smaller than 10% of the population.

If these conditions are met then

$$\hat{p} \sim N\left(p, \frac{p(1-p)}{n}\right)$$

Note that if third condition is not met, then the standard error can be adjusted.

1.2 Confidence Interval

Example

According to a Gallup Survey¹ of 1017 adults living in US 66% of Americans favor legalizing marijuana. Compute 95% confidence interval for the population proportion of those who favor legalizing marijuana.

¹<https://news.gallup.com/poll/267698/support-legal-marijuana-steady-past-year.aspx>

For the same problem above, calculate 90% confidence interval.

Interpretation of confidence intervals

1.3 Hypothesis Testing

Key points:

- We can never know the truth.
- In hypothesis testing, we assume the null is true.
- We never accept the null hypothesis. We fail to reject it.
- The data can provide evidence against the null and we may reject the null.

Example: In a survey of 1520 adults Gallup ² asked opinions on immigration. Each immigration item was asked to only randomly selected half of the sample. The survey results state that 41% favor expanding construction of walls along U.S.-Mexico border. Conduct a hypothesis test to evaluate whether the majority favors or opposes the wall.

²<https://news.gallup.com/poll/235775/americans-oppose-border-walls-favor-dealing-daca.aspx>

Example: *Do you favor or oppose the death penalty for persons convicted of murder?* was a question asked on General Social Survey 2018. Of those who responded with a choice, 1385 favored and 808 opposed. Conduct a hypothesis test to evaluate whether majority of Americans favor capital punishment for persons convicted of murder.

Decision Errors

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	Correct Decision	Type I Error (α)
	H_A true	Type II error (β)	Correct Decision

Example

2 Inference for a Difference of Two Proportions

2.1 Conditions

1. The data are independent within two groups and between two groups.
2. Each group needs to have 10 successes and 10 failures.

2.2 Confidence Interval

2.3 Hypothesis Testing

If these conditions are met then

$$(\hat{p}_1 - \hat{p}_2) \sim N\left(p_1 - p_2, \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)$$

Example: Does taking a college level science class change views on after life? Below are the responses from General Social Survey in 2018.

		Belief in Life After Death	
		Yes	No
College Science Course	Yes	375	75
	No	485	115

Example: Calculate the 95% confidence interval for the difference in proportion of belief in after life between those who have taken a college science course and those who have not in the population

Example: Evaluate whether taking a college level science class change views on after life with a hypothesis test.